

# ***MD simulations of diffusion, sputtering and bubble formation in liquid Li***

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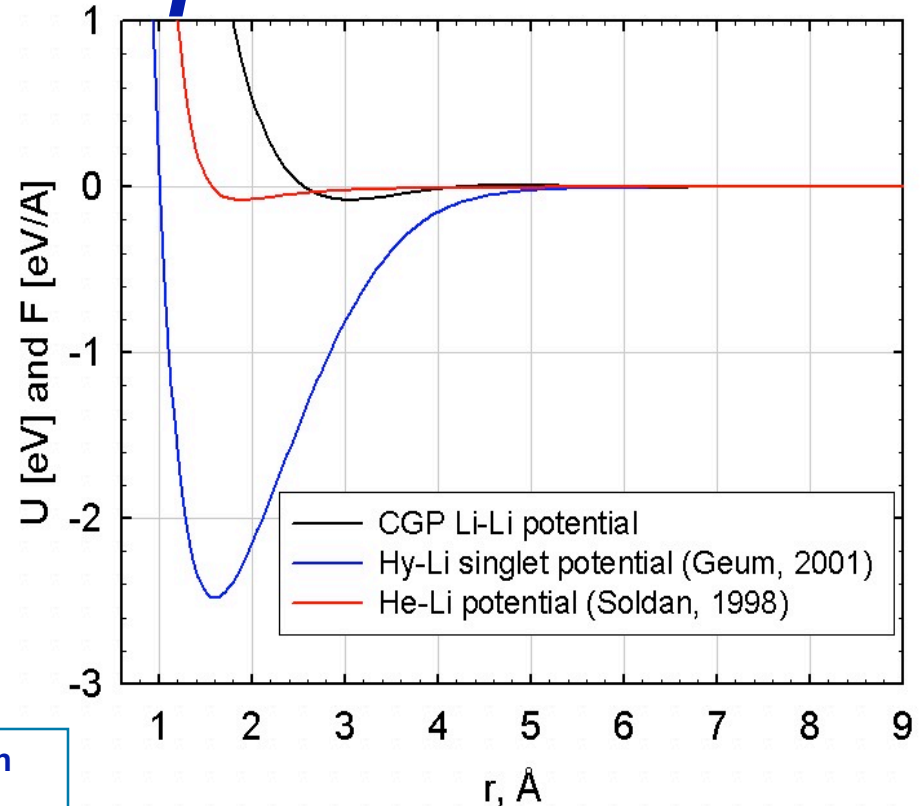
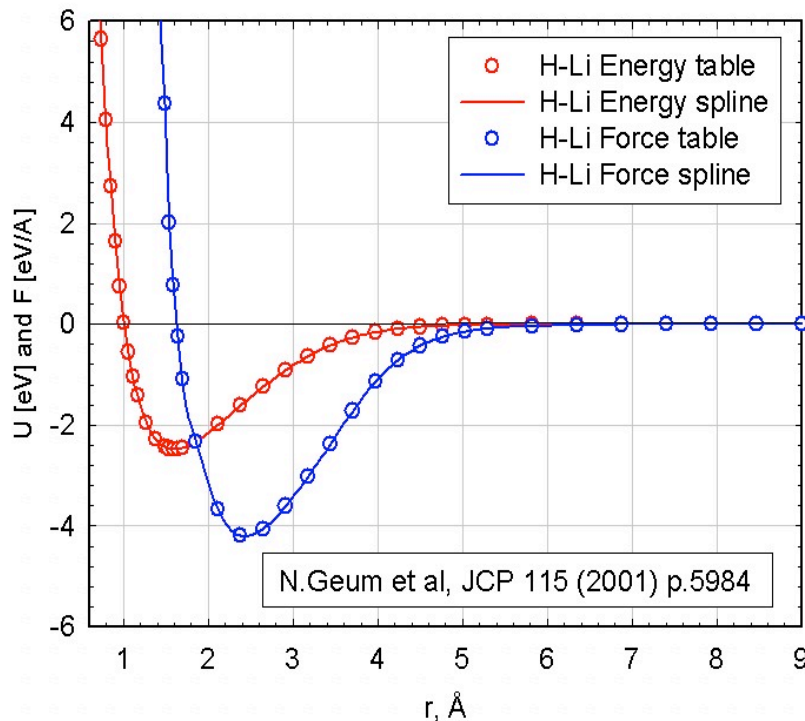


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# *Outline*

- ☐ Introduction – H, Li potentials
- ☐ Diffusivity of H,D,T atoms in Lithium
- ☐ Sputtering of Li by H, D, T ions
- ☐ Bubble formation in liquid Li

# H/Li interaction potential



MD method needs accurate interaction potentials between the particles in the system.

Recently, a new H-Li potential was developed by ab-initio calculations of the lowest singlet  $^1S$  and triplet  $^3S$  states for LiH and applied for analysis of trapping of cold atoms in [1].

The singlet form of this H-Li potential (circles - tabulated data) were used for building the energy and force cubic splines (red and blue lines).

[1] N. Geum et al, JCP115 (2001) p.5984

H-Li potential well (blue line) is much deeper than the He-Li (red line) and Li-Li potential (black line).

# Analytical EAM-Li-Li potential #1

TB QM (Eq.1) was used in [1] for obtaining a suitable ion-ion potential for disordered Lithium systems (small clusters, surfaces).

$$(\varepsilon_{ia}^0 - E) |ia\rangle + \sum_{j \neq i} \sum_b t_{ab}^{ij}(r_{ij}) |ib\rangle = 0, \quad (1)$$

$$U_{\text{coh}} = U_{\text{el}} + U_{\text{rep}},$$

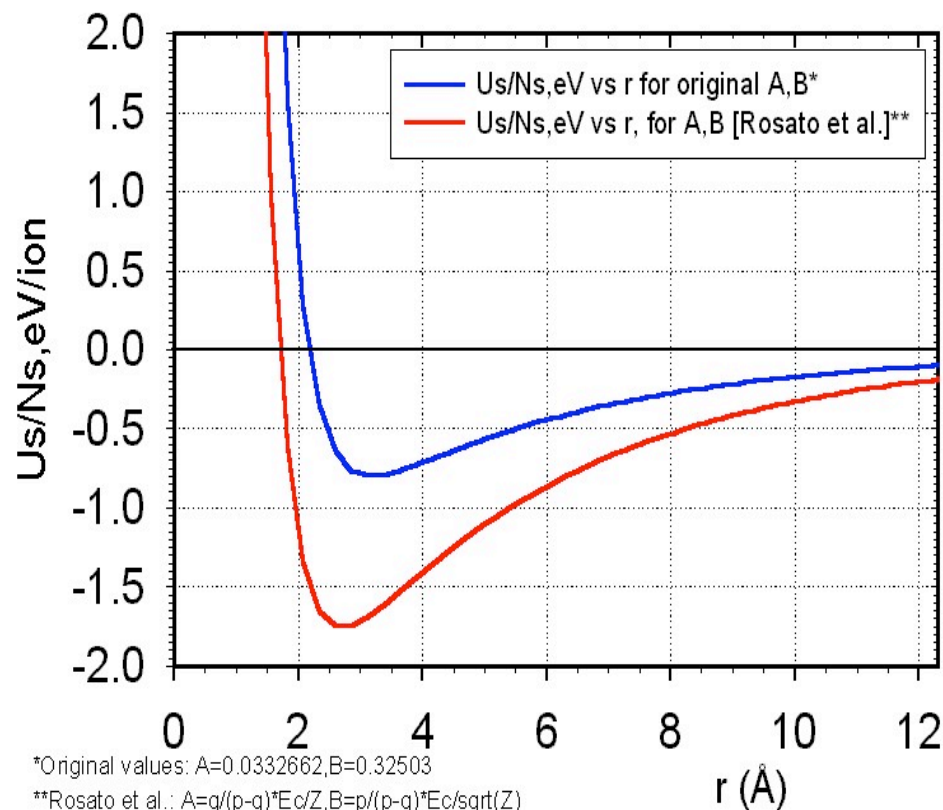
$$U_{\text{rep}} = \sum_i \varepsilon_0 \sum_{j \neq i} \exp \left[ -p \left( \frac{r_{ij}}{r_0} - 1 \right) \right],$$

$$U_{\text{el}} = - \sum_i \left\{ \sum_{j \neq i} \zeta^2 \exp \left[ -2q \left( \frac{r_{ij}}{r_0} - 1 \right) \right] \right\}^{1/2}$$

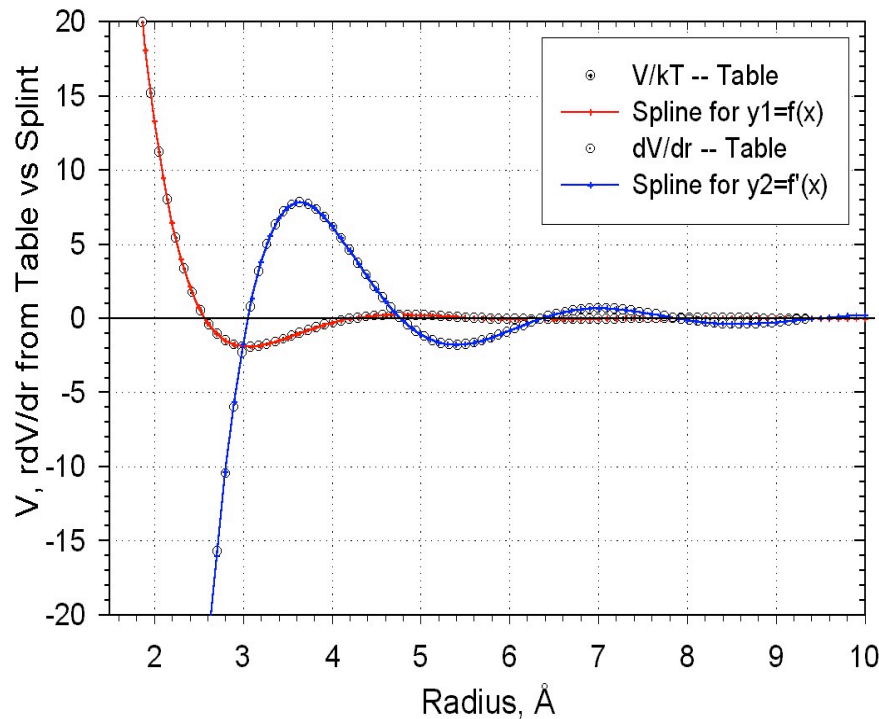
## Parameters used for this simulation:

Li-Li:	$\varepsilon_0$ , mRy	$\zeta_0$ , mRy	p	q	$r_0$ , a.u.
	2.4450	23.889	7.75	0.737	5.490

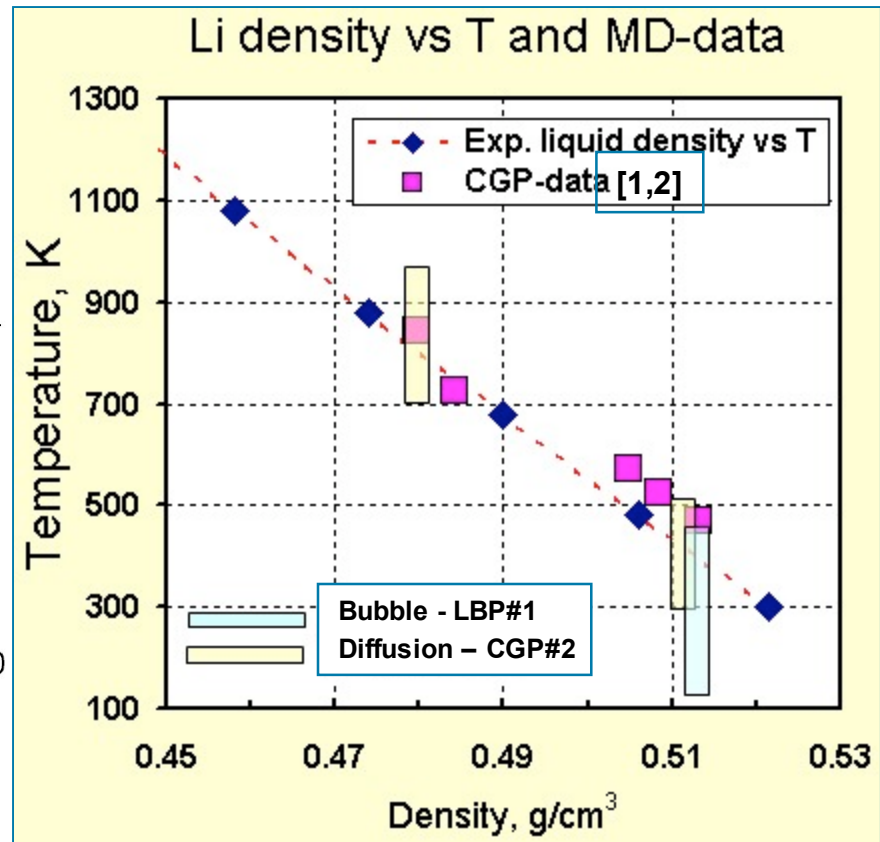
[1] Y. Li et al, Phys. Rev. B57 (1998) 15519



## Li-Li bulk interaction potential #2



The tabulated Li-Li bulk potential and its first derivative given in [1,2] (circles) and cubic splines (red and blue lines) used in this work.

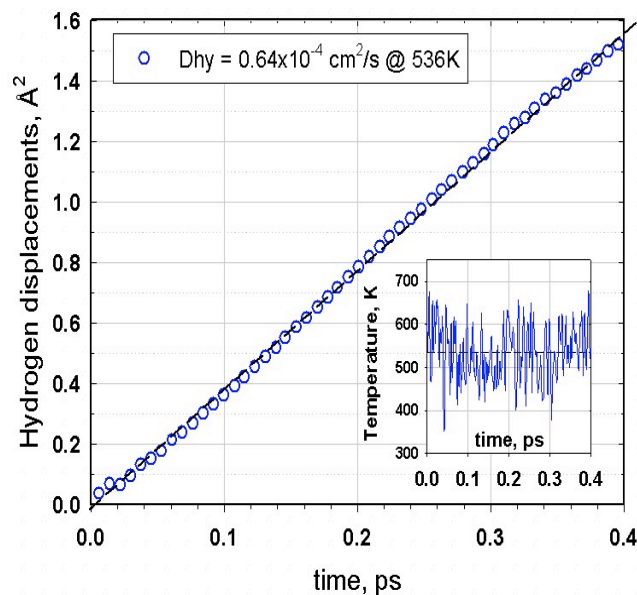
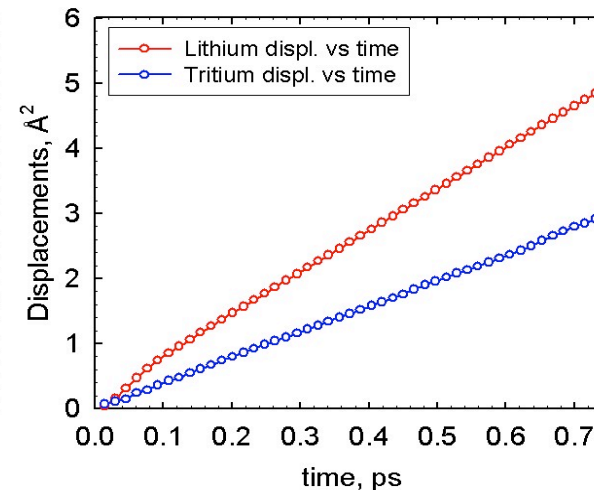
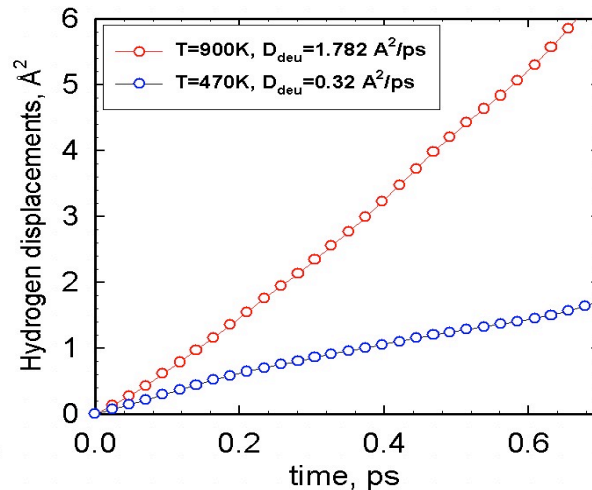
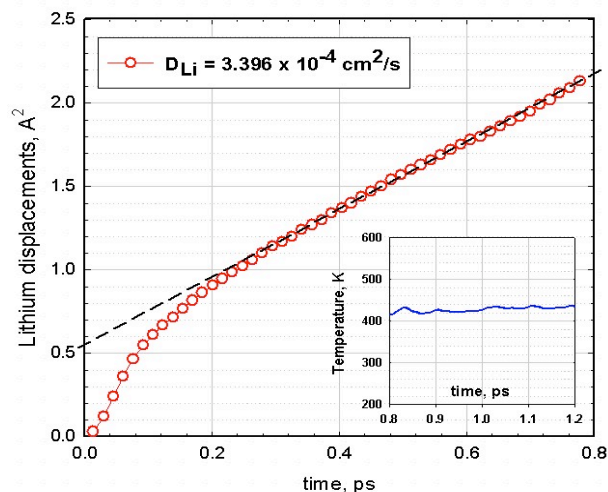


### References:

- [1] M. Canales et al, J. Phys. 5 (1993) 3095.
- [2] M. Canales, Phys.Rev. E50 (1994) 3656.
- [3] H. Morimoto et al, NASDA reports, 1999.



# Li, H, D, Tr displacements

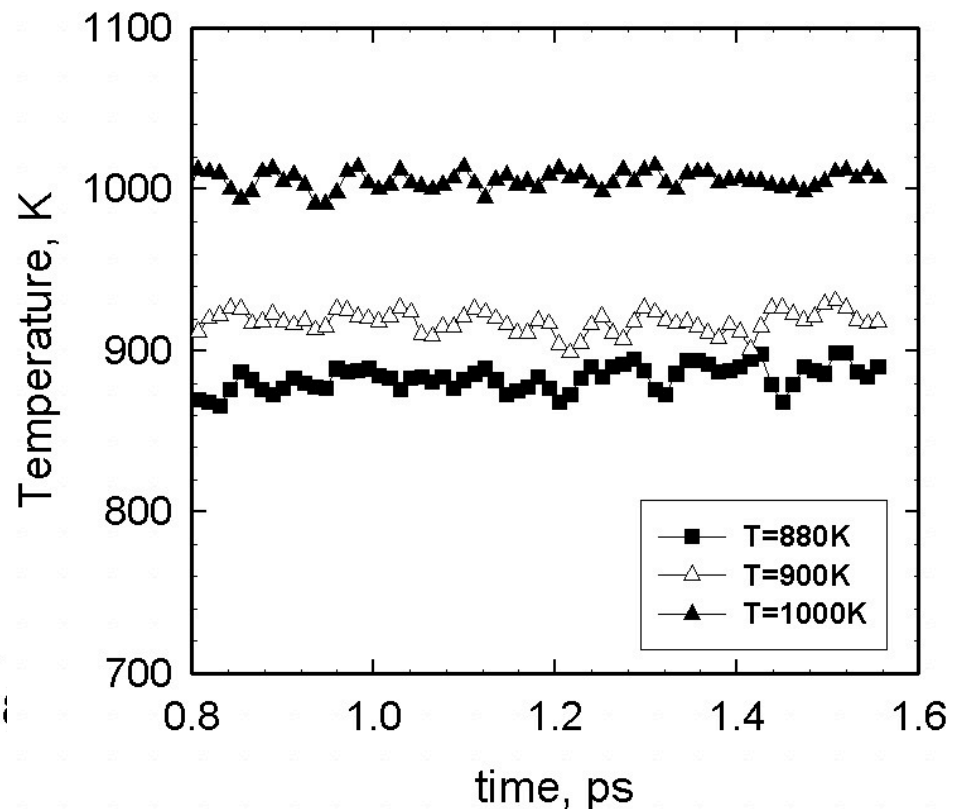
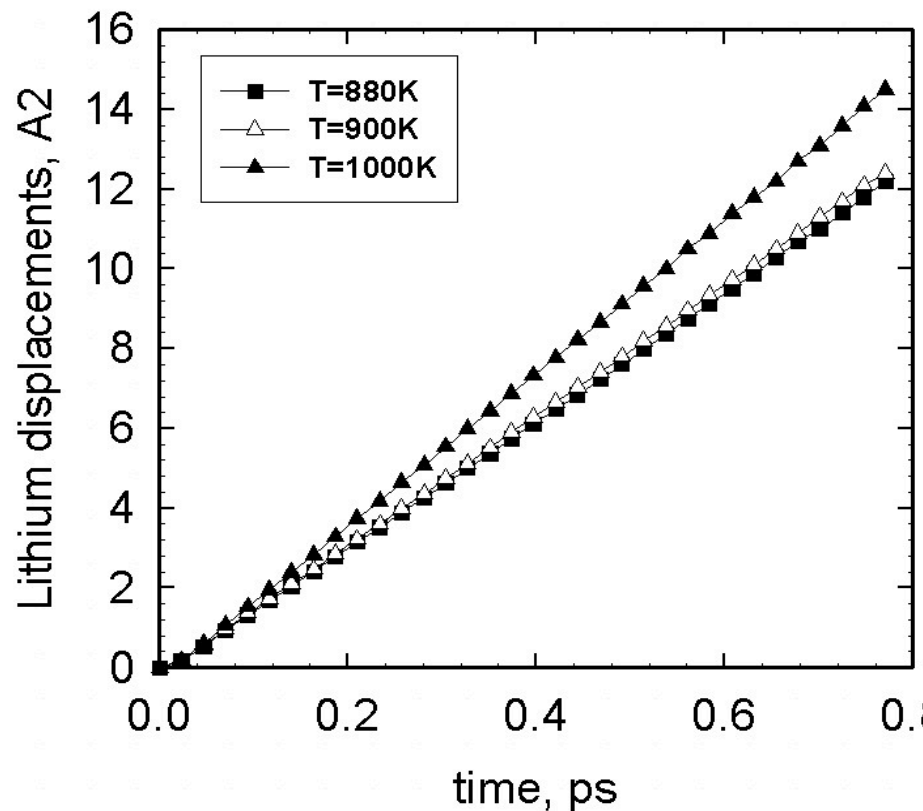


$$\langle \Delta r^2(t) \rangle = \left\langle \frac{1}{N} \sum_{i=1}^N \vec{r}_i(\tau) \vec{r}_i(\tau + t) \right\rangle_{\tau} \quad (1)$$

Time-dependence of the mean-square-displacements of lithium ions and hydrogen, deuterium and tritium atoms were calculated by (eq.1) at various temperatures and densities of a mixture of Li and H (< 1% concentration in lithium).



# Liquid Li displacements

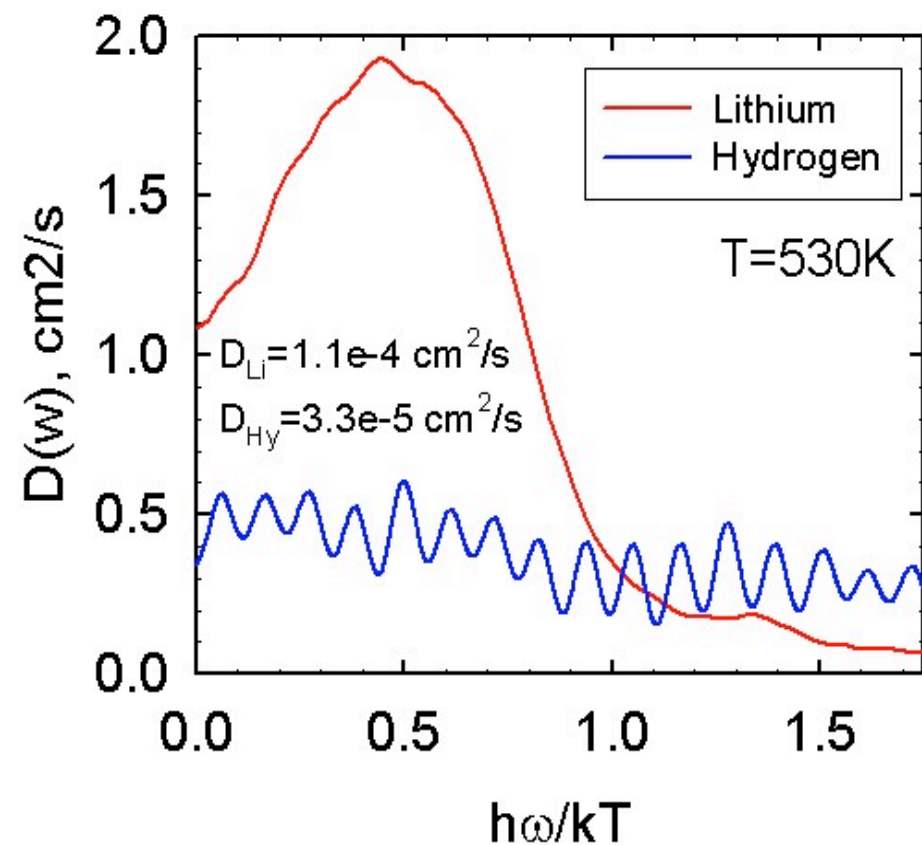
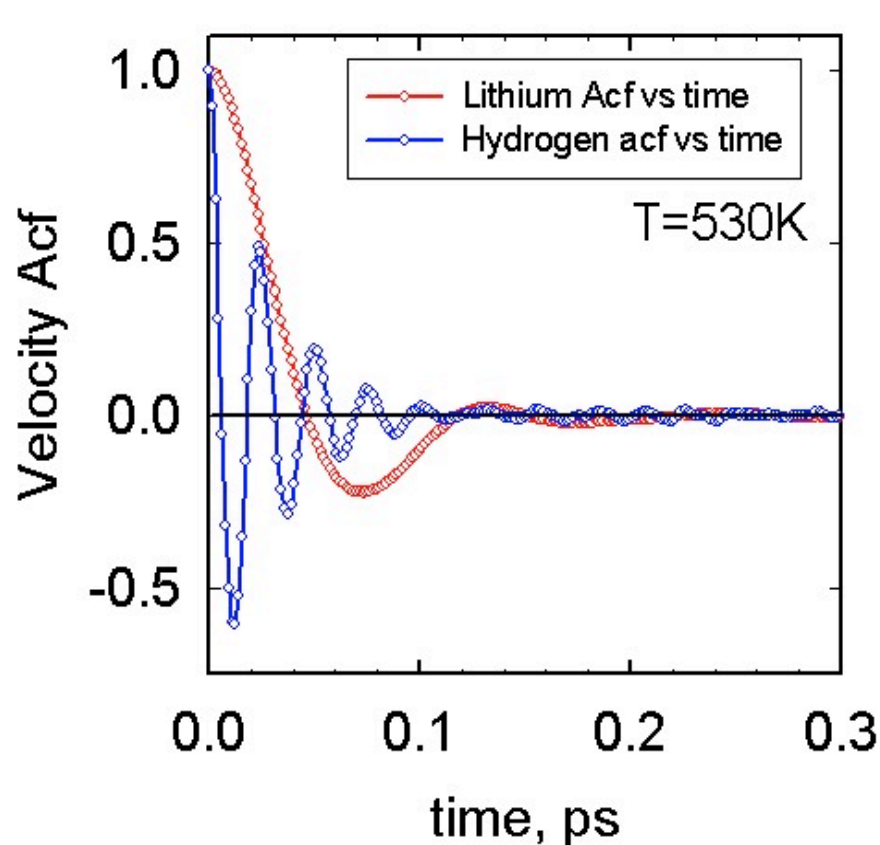


$$(2) \quad D_{\alpha} = \lim_{t \rightarrow \infty} \frac{\langle \Delta r_{\alpha}^2(t) \rangle}{6t},$$

$$\alpha = Li, H, D, T$$

The diffusion coefficients were obtained by two methods: a) 1<sup>st</sup> method uses Eq. 2 which obtains diffusion coefficient as a tangent of the linear region of the displacement function; b) The 2<sup>nd</sup> method consists in calculation of the velocity auto-correlation functions  $Acf(t)$  and then in obtaining the spectral density  $D(\omega)$ . The diffusion constants are the values at  $\omega=0$ .

# Li, H Acf and diffusivity

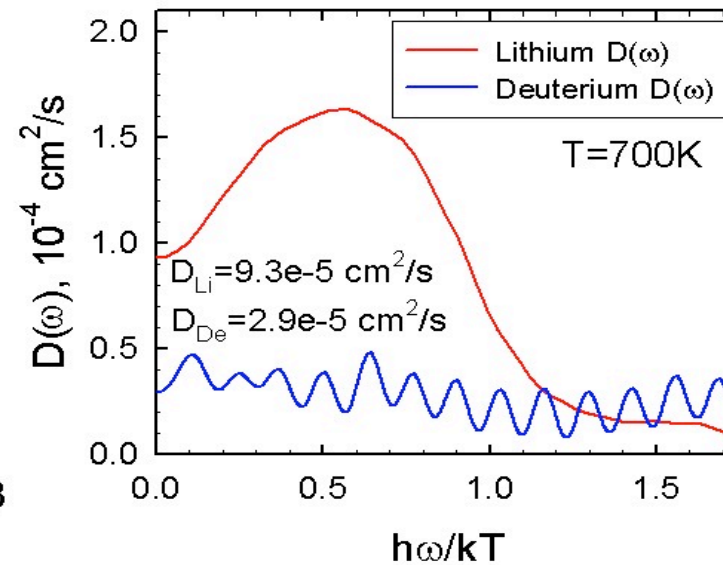
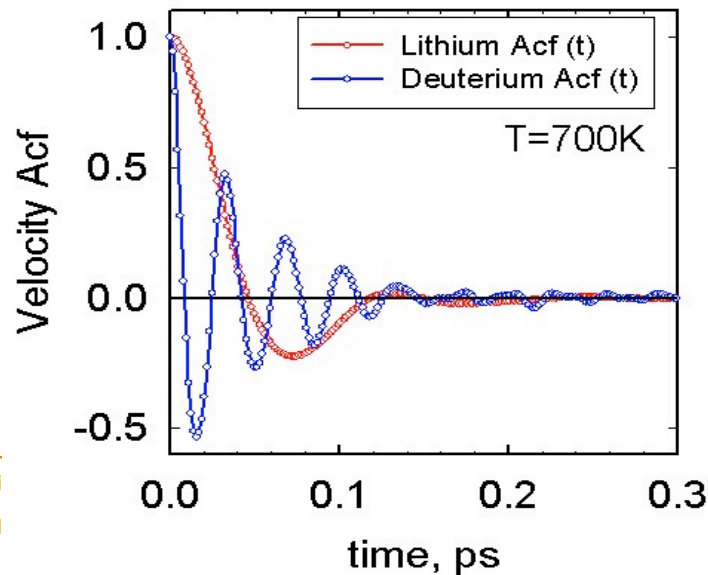
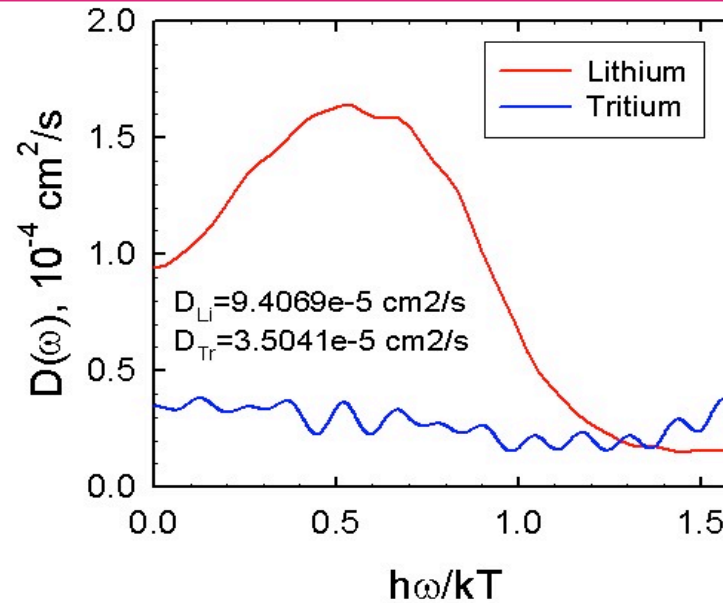
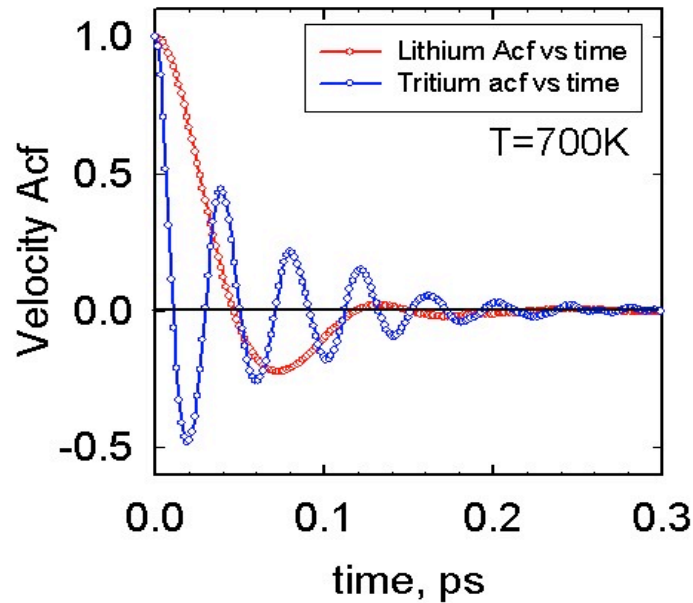


$$(3) \quad Acf(t) = \left( \frac{m}{3k_B T} \right) \sum_{i=1}^N \langle \vec{v}_i(0) \vec{v}_i(t) \rangle_{\tau}$$

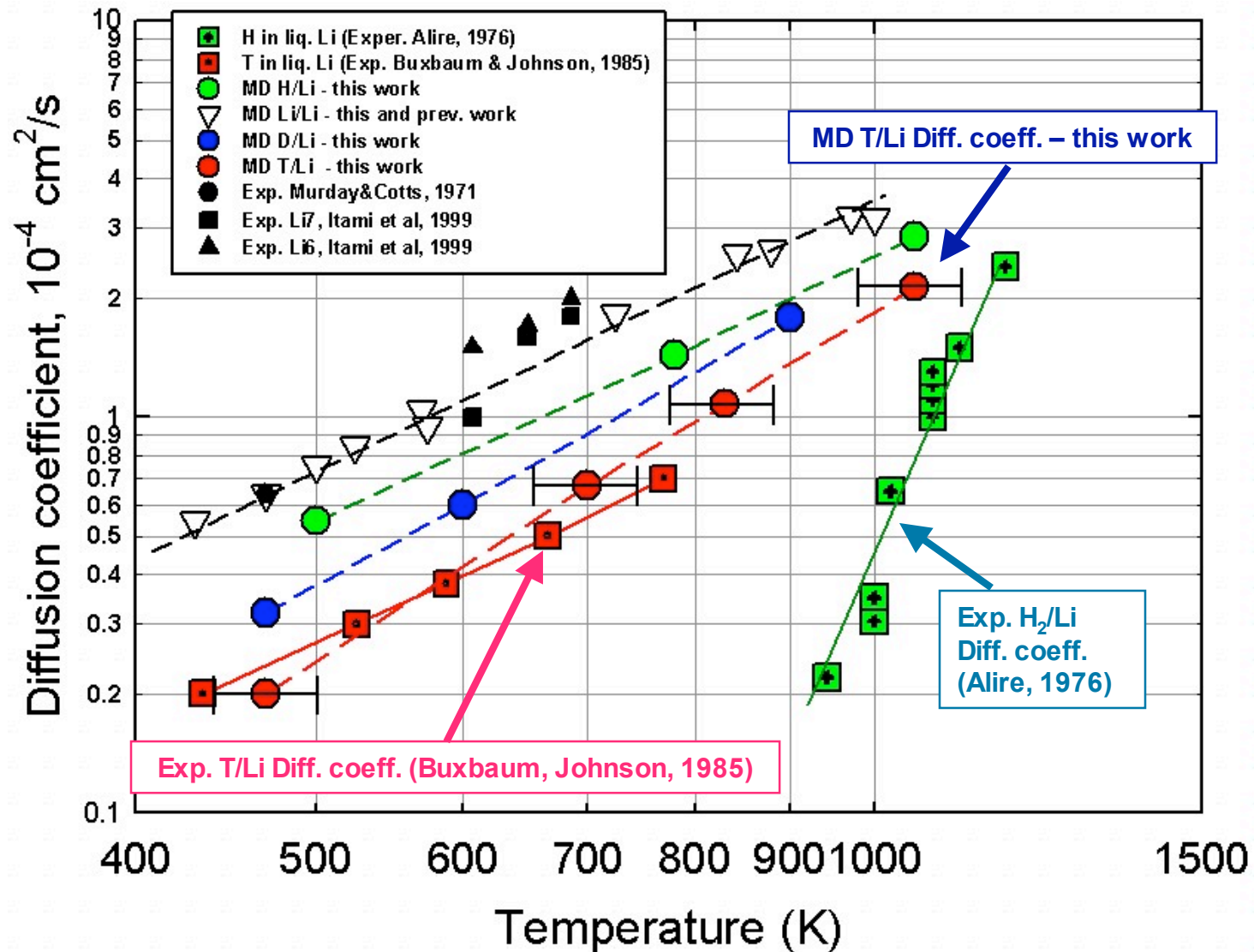
$$(4) \quad D(\omega) = \left( \frac{k_B T}{m} \right) \int_0^{\infty} Acf(t) \cos(\omega t) dt$$



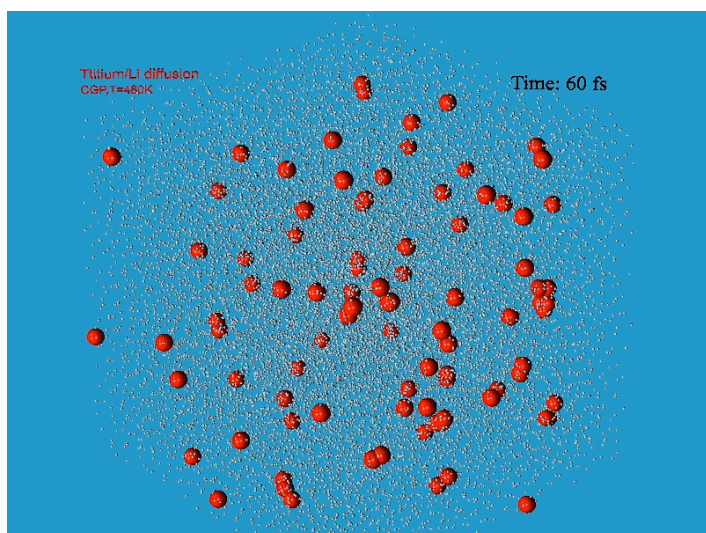
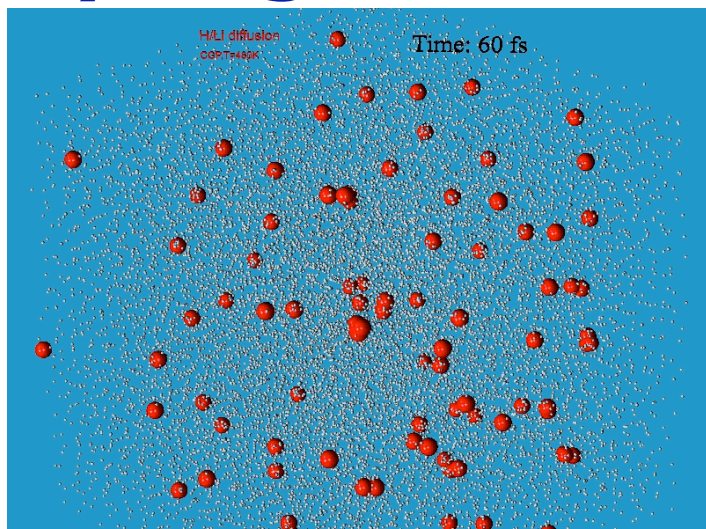
# ***D, Tr, Li acf and diffusivities***



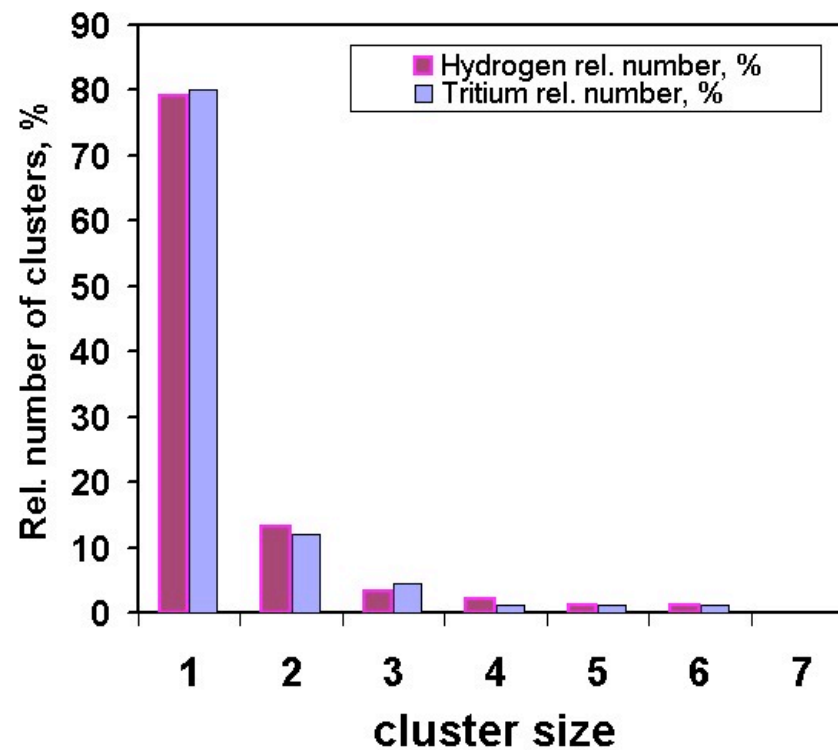
# Diffusivity of H, D, Tr in liquid Li



# Hydrogen cluster formation in liquid Li



n	1	2	3	4	5	6	7
Nn (Hydrogen)	72	12	3	2	1	1	0
Relative number of Hn, %	79.1	13.2	3.3	2.2	1.1	1.1	0
Nn (Tritium)	73	11	4	1	1	1	0
Relative number of Trn, %	80.2	12.1	4.4	1.1	1.1	1.1	0

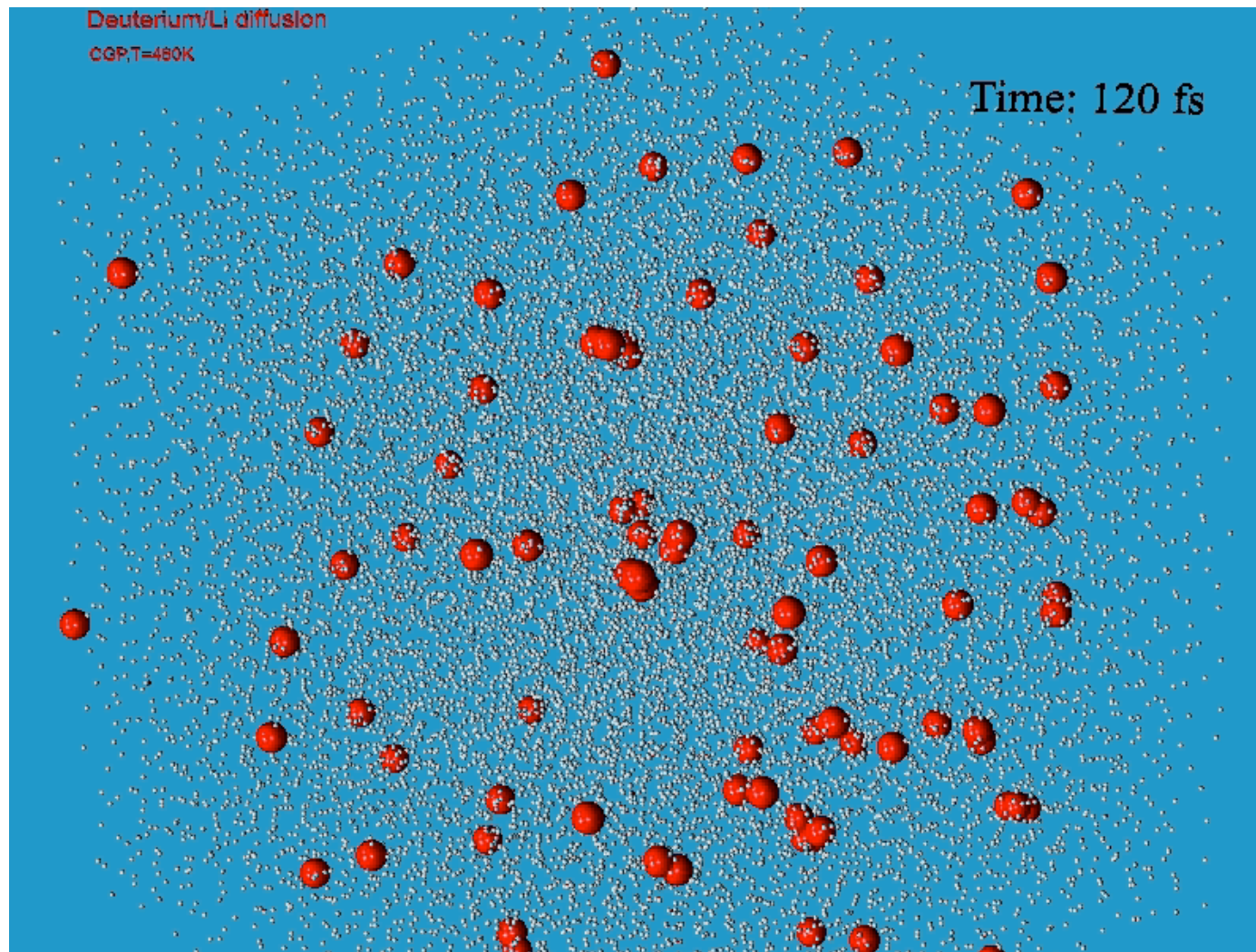


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# *Deuterium clusters in liquid Li @ 480K*



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# Summary #1

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- The H-Li potential was tested by calculating the diffusion coefficients of Hydrogen isotopes in liquid Li at various temperatures and densities.
- The calculated tritium diffusion coefficient in liquid Lithium is in good agreement with the experimental results of Buxbaum & Johnson (1985).
- A noticeable amount of hydrogen clusters in liquid Li (>10%) is obtained by this MD study. These clusters are still highly mobile.
- Future MD tasks: a) Temperature stabilization during cluster formation; b) The kinetics of H<sub>n</sub> cluster formation could be studied by direct MD.

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# *Outline*

- ☐ Diffusivity of H, D, T atoms in liquid Lithium
- ☐ **Sputtering of Li by H,D,T ions**
- ☐ Bubble formation in liquid Li



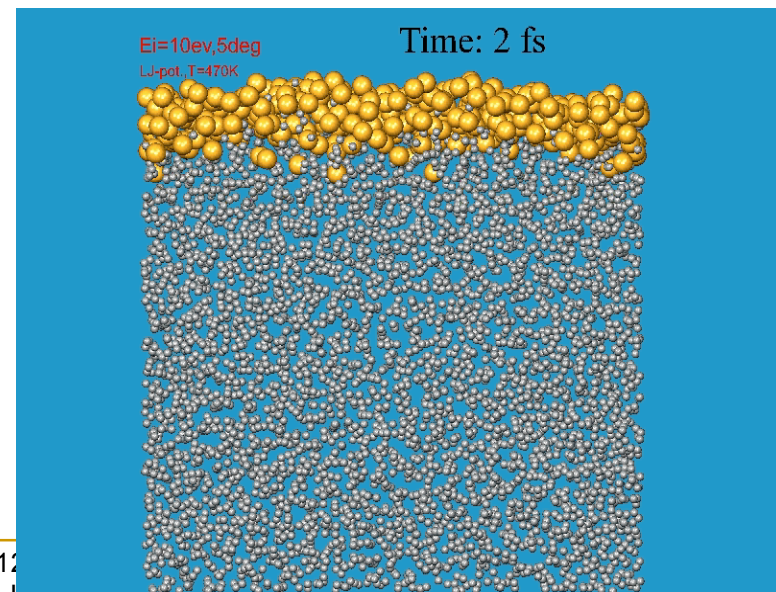
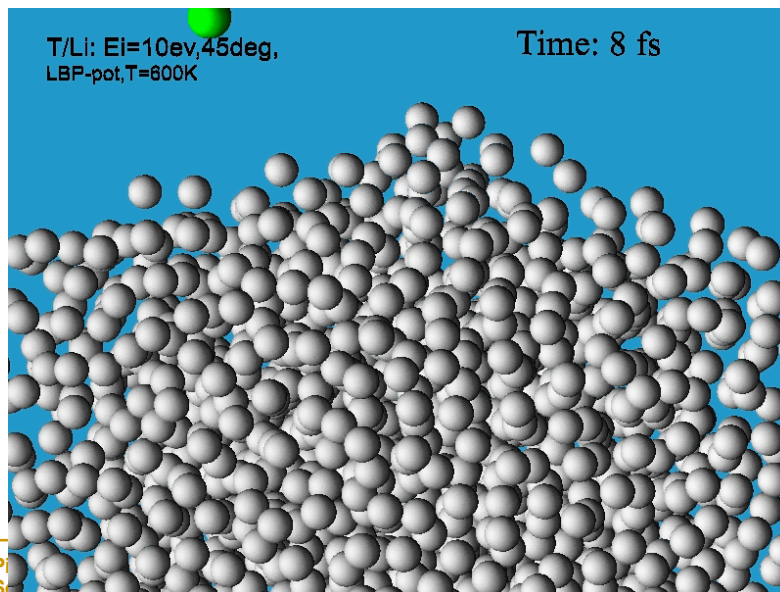
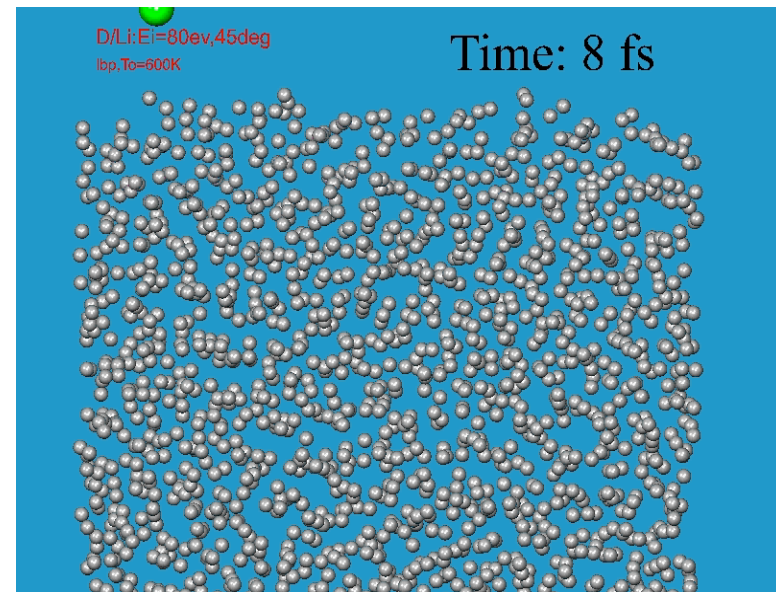
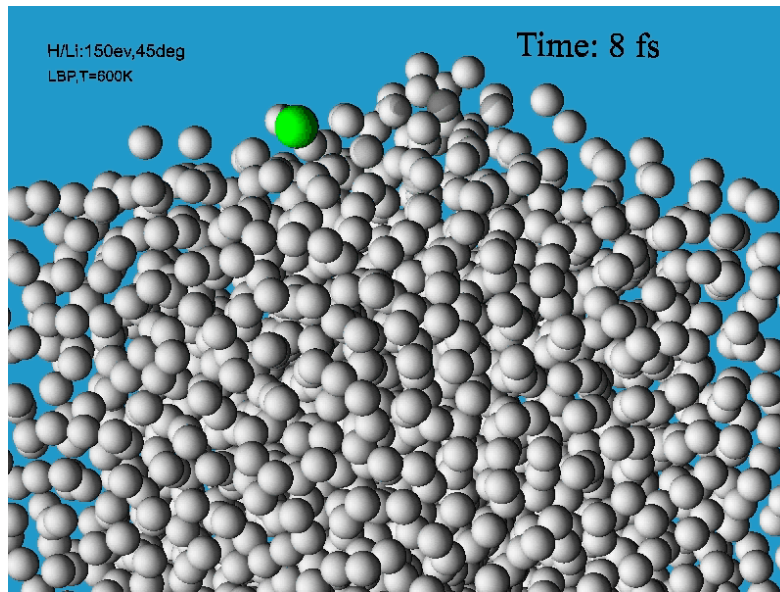
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# *Sputtering simulation model*

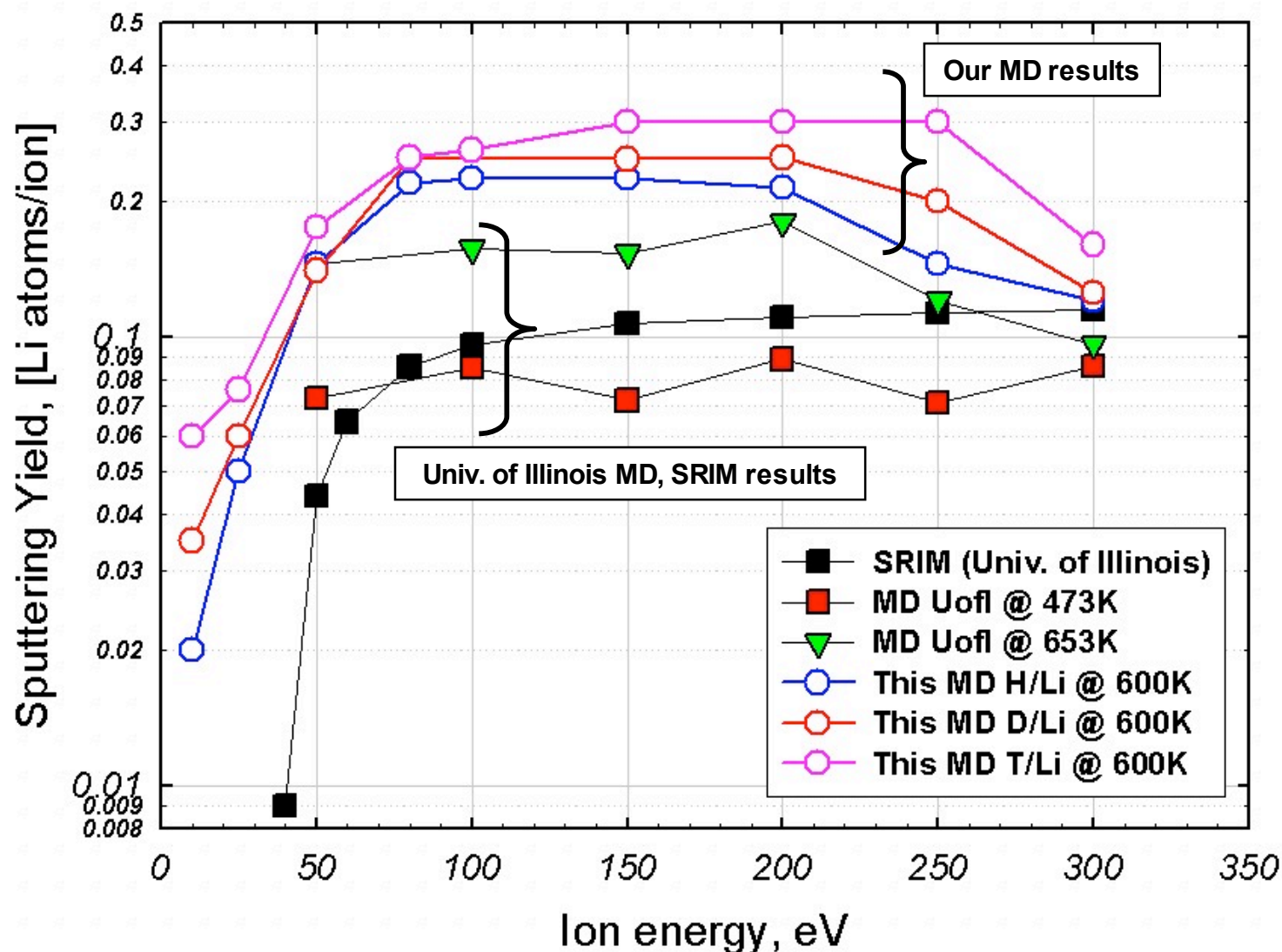
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- The singlet-type H-Li potential given by Geum et al, 2001 was used for simulation of H,D, and T collisions with a lithium surface.
- Lithium surface was modeled by various types of Li-Li potentials.
- Image forces were added to the interaction between Hydrogen ions with the liquid Lithium surface.
- Sputtering yield, reflection and sticking coefficients, energies etc. were calculated for a wide range of temperatures and ion energies in the interval of 10 – 300 eV.

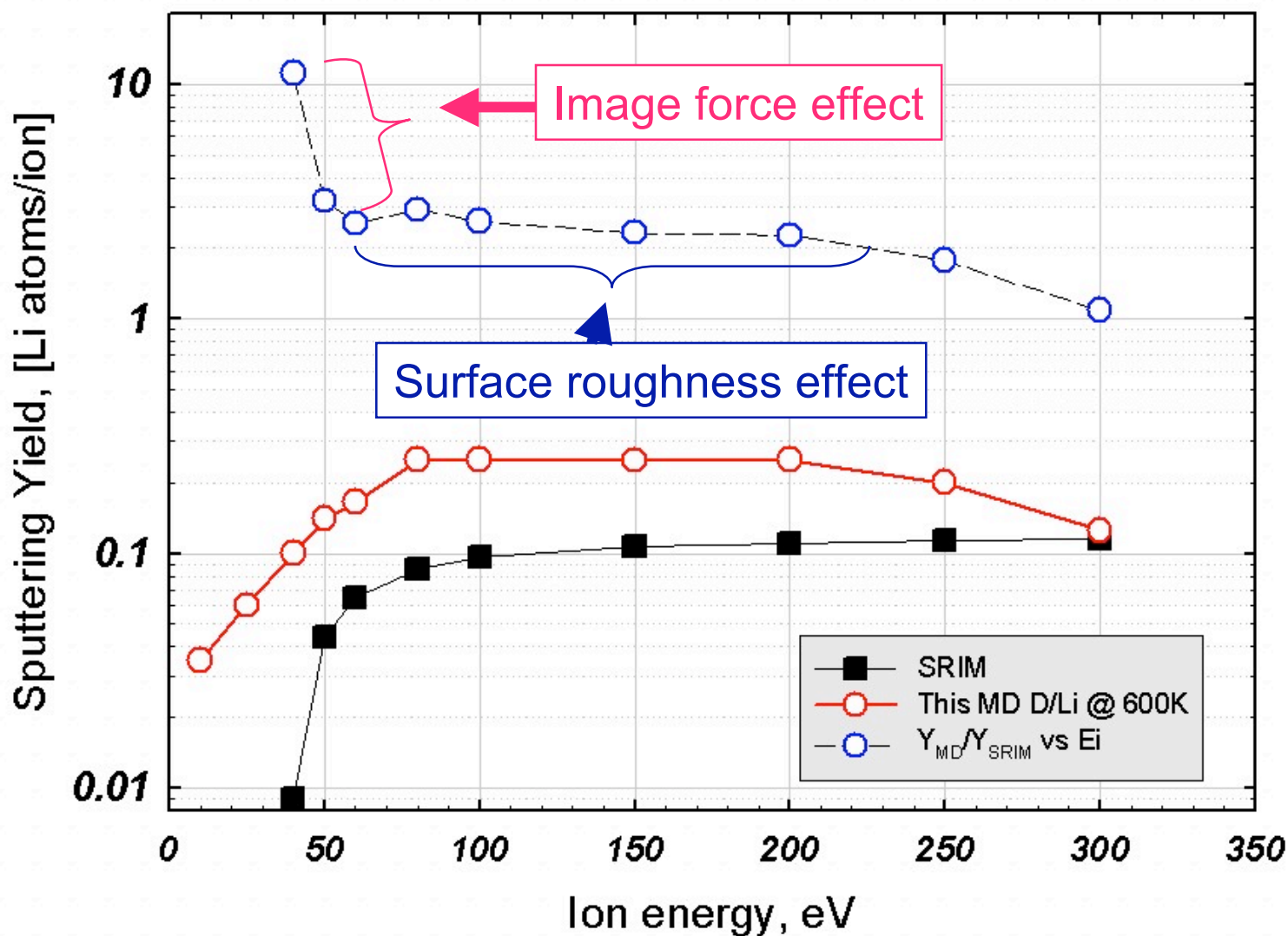
# *H-Li collision movies*



# Sputtering yields of Li surface with H, D, Tr ions



## Comparison of D/Li sputt. yields: MD vs SRIM





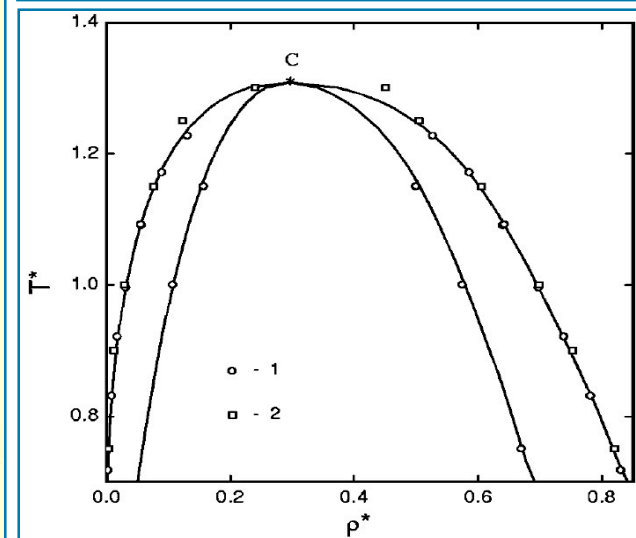
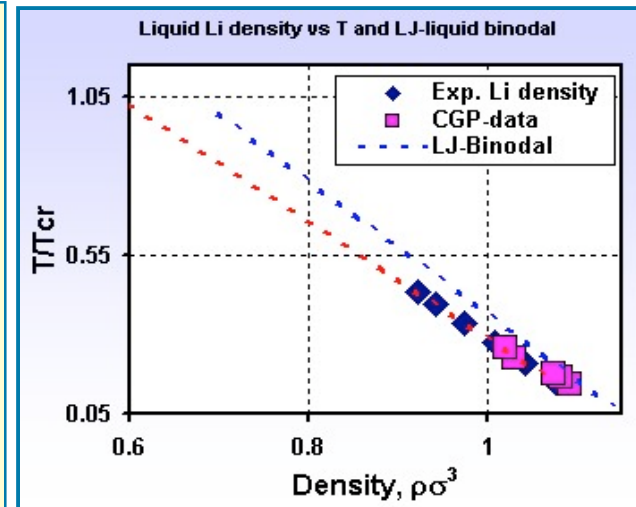
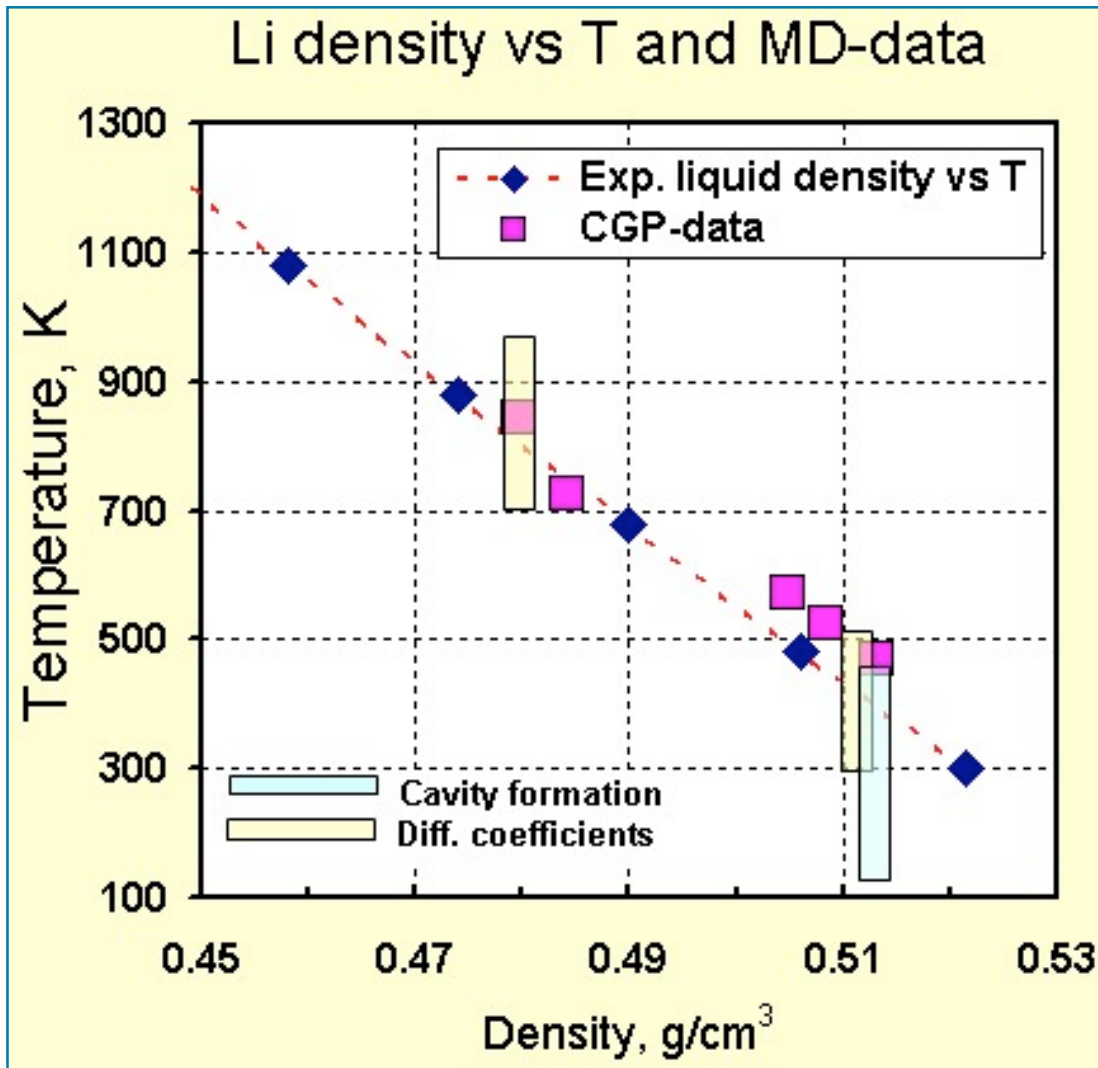
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# *Outline*

- ☐ Diffusion coefficients of H, D, T in liquid Lithium
- ☐ Sputtering Yields of H, D, T on a Li surface
- ☐ **Bubble formation in liquid Li**

# Liquid Li density vs Temperature





# Surface tension and free energy of a cavity

The Tolman's formula

$$\gamma_s / \gamma_\infty = 1 - \frac{2\delta}{R_s},$$

$$\delta = R_e - R_s$$

$\gamma_s$  – Bubble surface tension,

$\gamma_\infty$  – plane surface tension,

$R_e, R_s$  – radii for the equimolar surface and the surface of tension

$$\Delta U(r) = -\frac{4}{3}\pi r^3 \cdot \Delta\omega + 4\pi r^2 \cdot \gamma_s,$$

$$\Delta\omega = P_g - P_l$$

$$\frac{d\Delta\Omega}{dr} = 0 \Rightarrow \Delta P = \frac{2\gamma_s}{R_s} \text{ (Laplace eq.)}$$

$$R_s = \frac{2\gamma_s}{P_l - P_g},$$

$$R_e = \frac{1}{\rho_l - \rho_g} \int_0^\infty r^3 \frac{d\rho}{dr} dr.$$

We obtained radial pressures around a bubble in liquid in analogy with the droplet capillary theory [1,2]. Bubbles in a LJ-system were studied by MD in [3].

## References

[1] Thompson, Gubbins et al, JCP, 1984

[2] Lee, Telo da Gama, Gubbins, JCP, 1986

[3] Park, Weng, Tien, Int. J. Heat & Mass Transfer, 12001



# MD calculation of variables

$$P(r) = P_N(r) + P_{Tr}(r)$$

$$P_T(r) = P_N(r) + \frac{r}{2} \frac{dP_N(r)}{dr},$$

$$P_N(r) = P_{Kin}(r) + P_U(r)$$

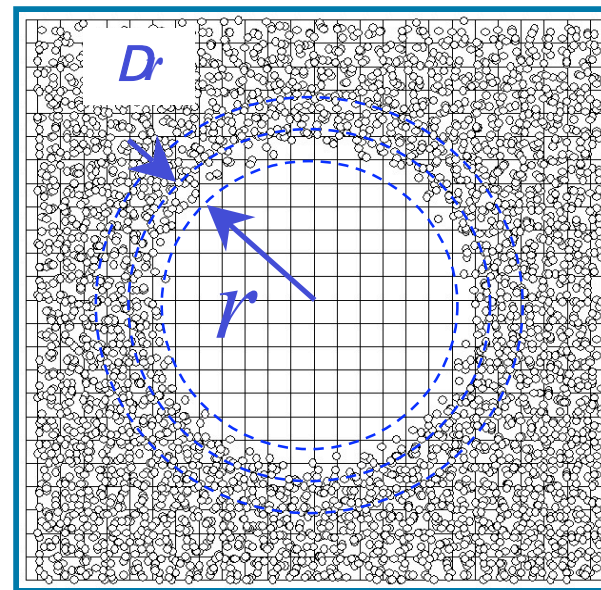
$$P_{Kin}(r) = \rho(r) k_B T,$$

$$P_U(r) = -\frac{1}{4\pi r} \sum_k \left| \vec{r} \cdot \vec{r}_{ij} \right| \frac{1}{r_{ij}} \frac{dU(r_{ij})}{dr_{ij}},$$

$$\gamma_S^3 = -\frac{1}{8} (P_l - P_g) \int_0^\infty r^3 \frac{dP_N}{dr} dr.$$

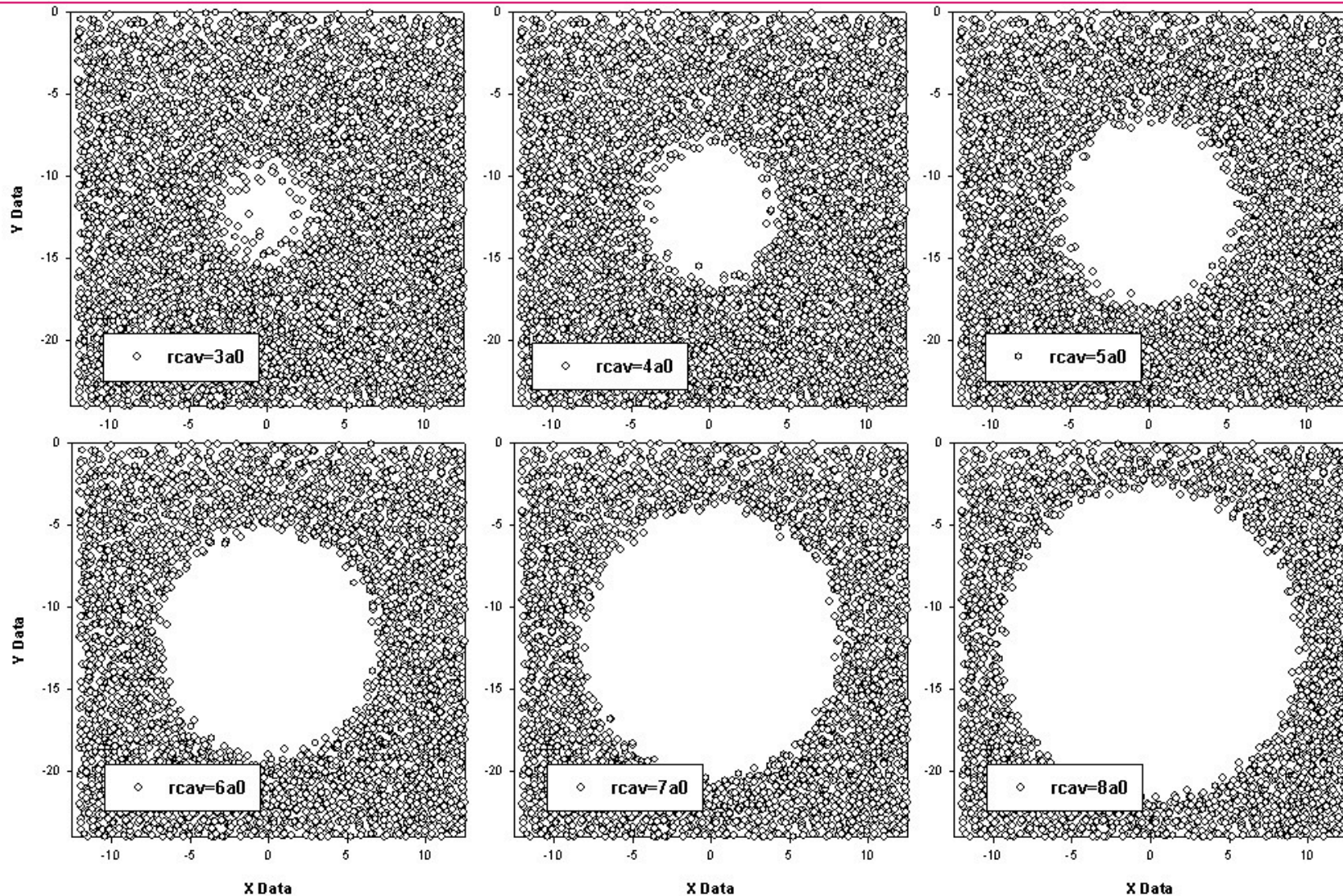
Average radial densities and pressures were calculated within a narrow spherical layers and compared with experimental data for plane surfaces (table).

$$\langle \rho(r_k) \rangle = \left\langle \frac{\Delta N}{\Delta r} \right\rangle_k, k = 1 - n$$



Metal	g, mN/m	g, [e/s <sup>2</sup> ]	g, [eV/Å]
Hg	500	0.264	2.226
Ga	750	0.396	3.339
In	550	0.290	2.448
K	100	0.053	0.445
Li	398	0.210	1.772

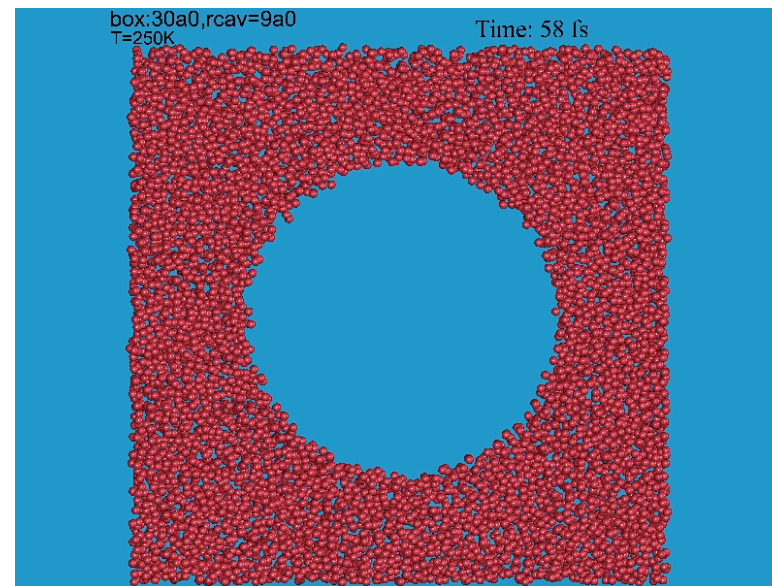
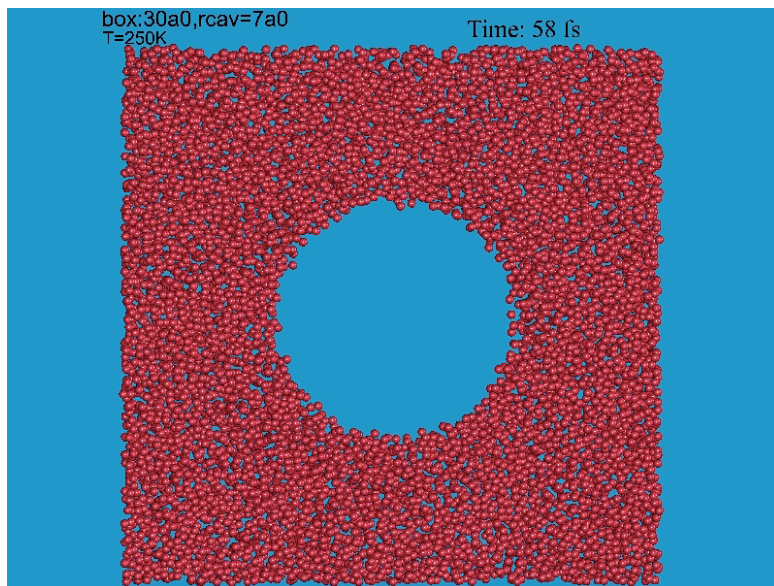
# Surface tension of cavity in Li



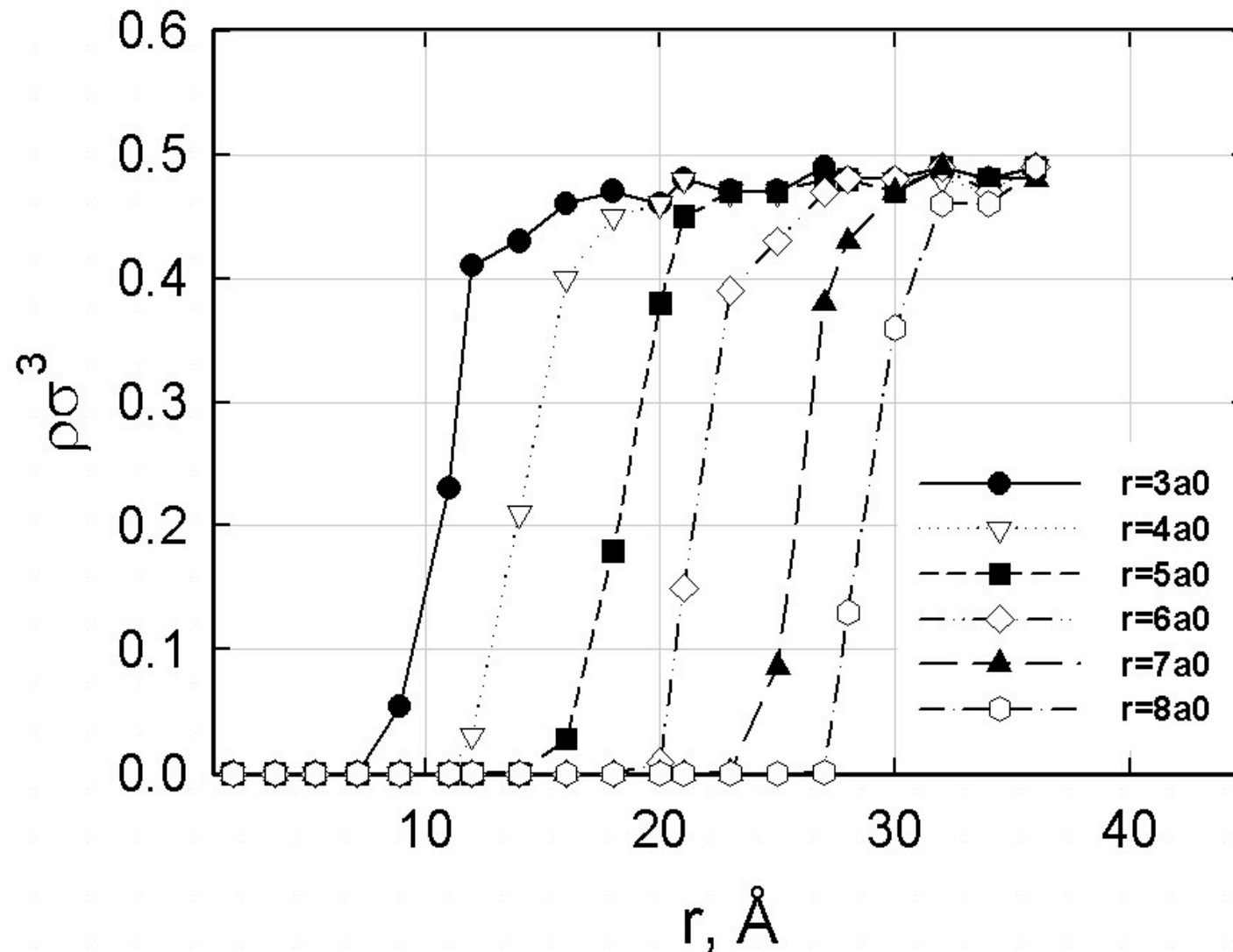
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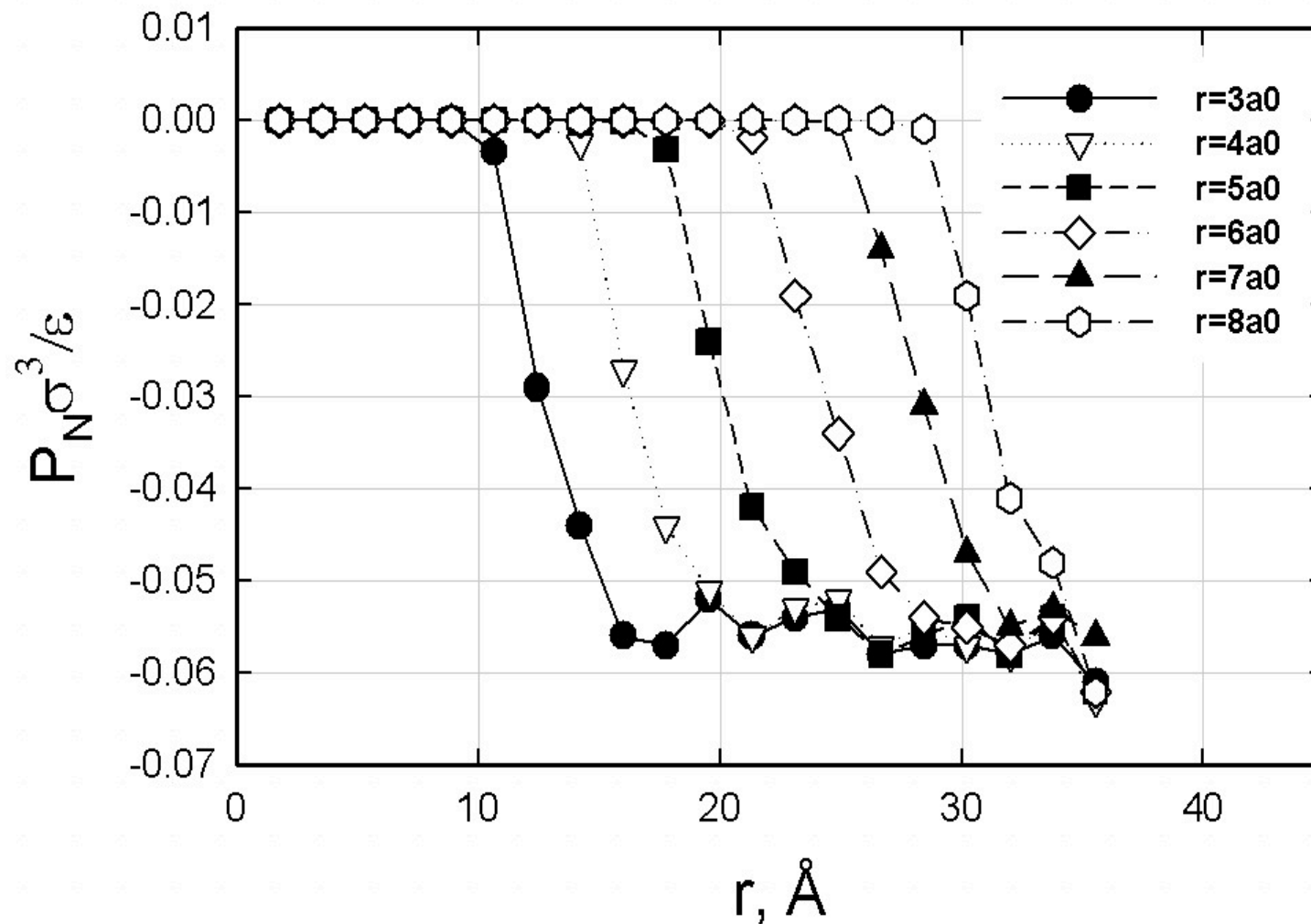
# *Cavity dynamics*



## Radial density around cavities in liquid Li



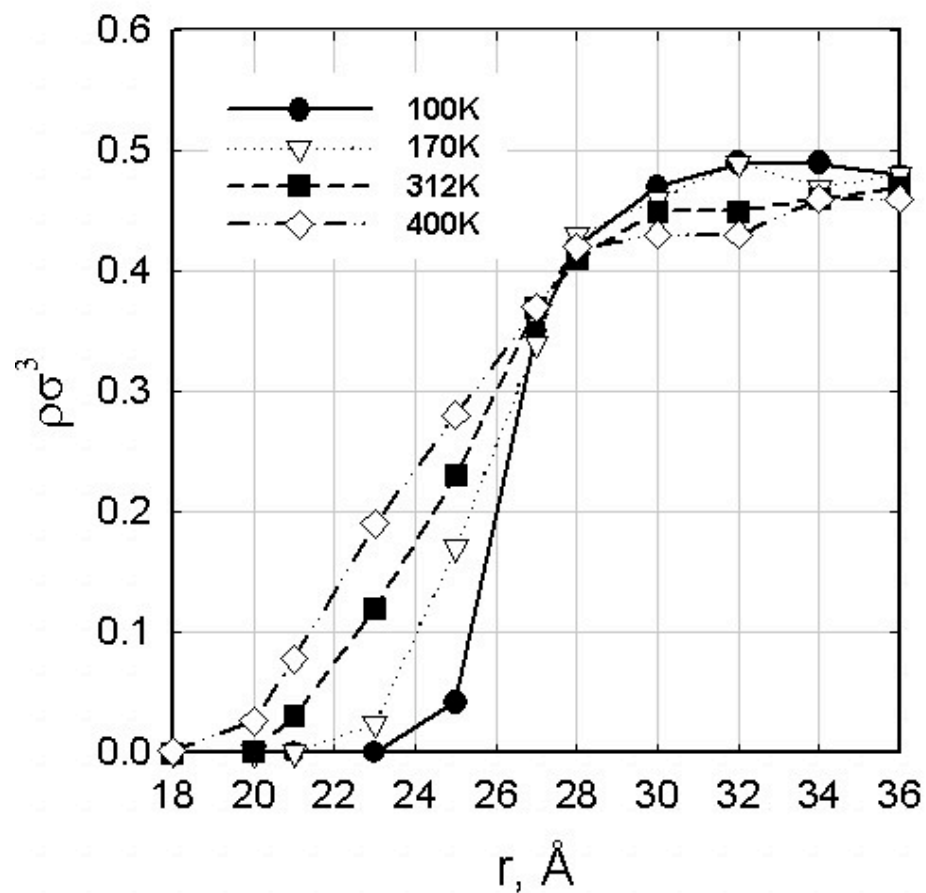
# Normal pressure in liquid Li



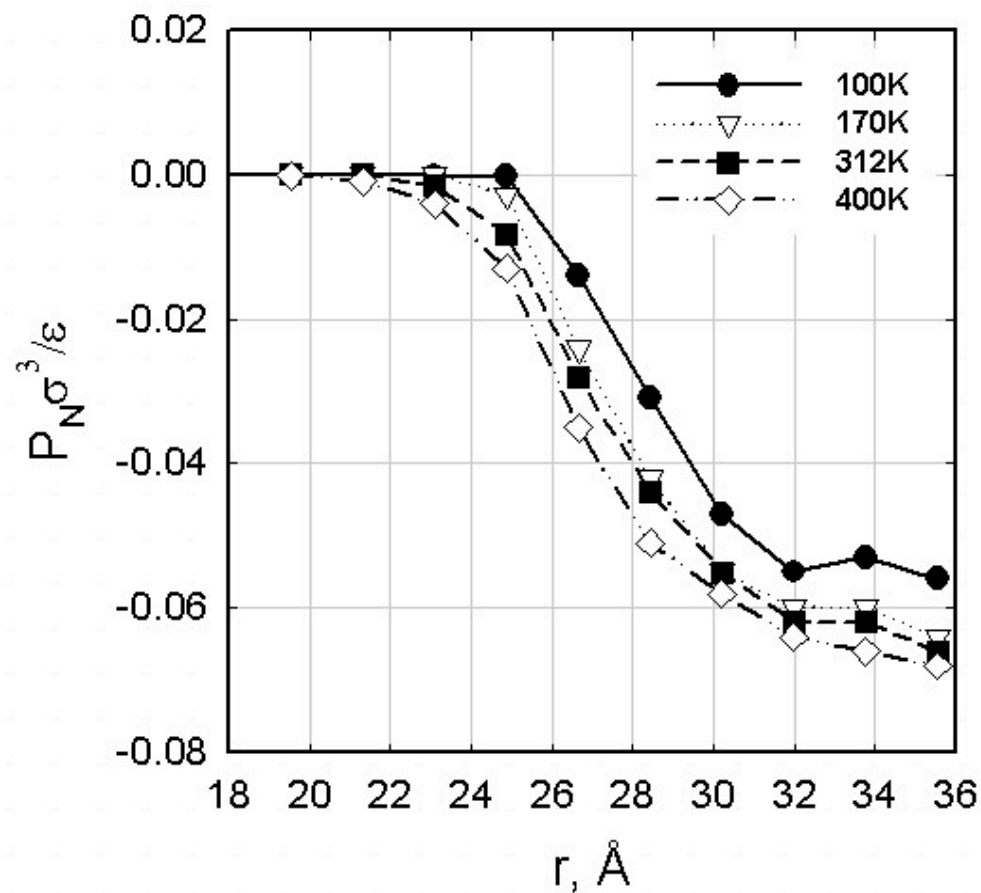


# Temperature dependence of $\rho$ , $P_N$

Radial density profiles for various bubble sizes

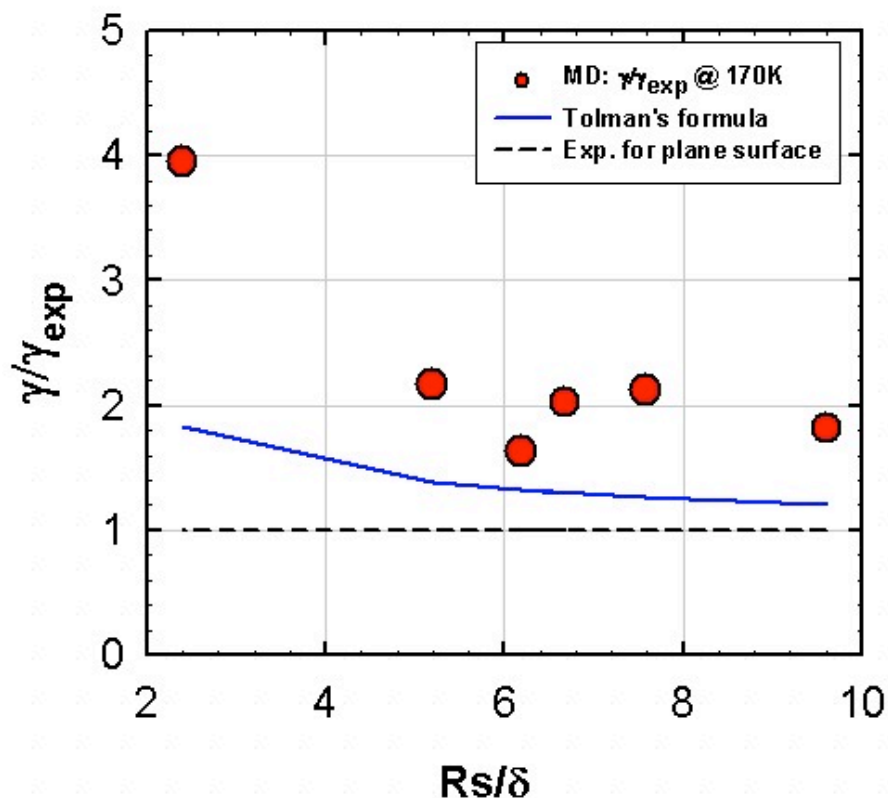


Radial normal pressure profiles

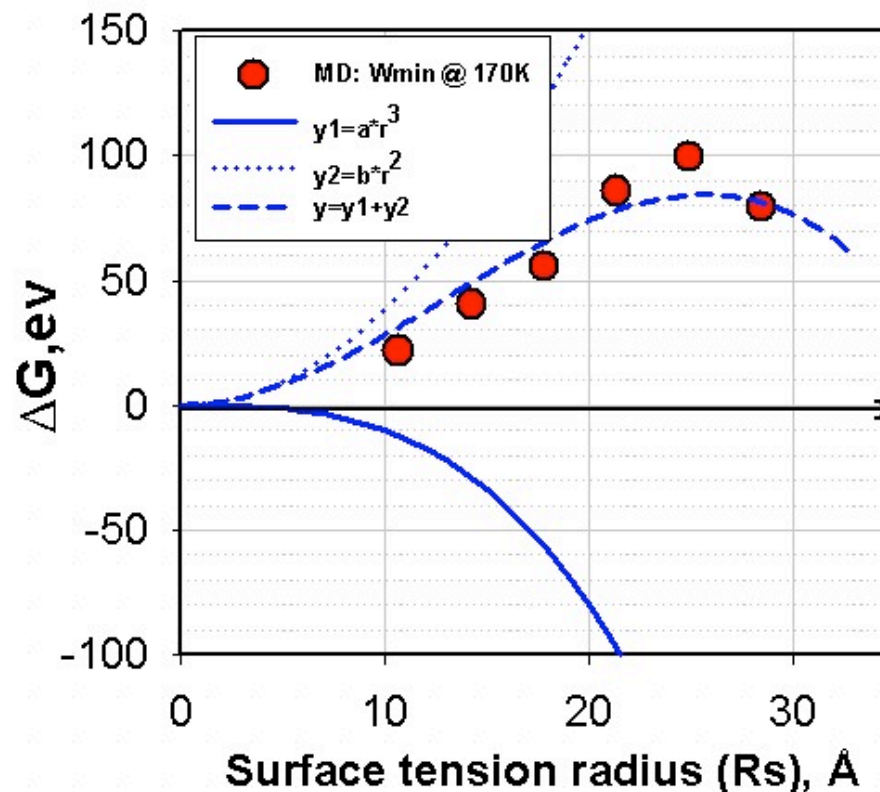


# Tolman's formula and Gibbs energy

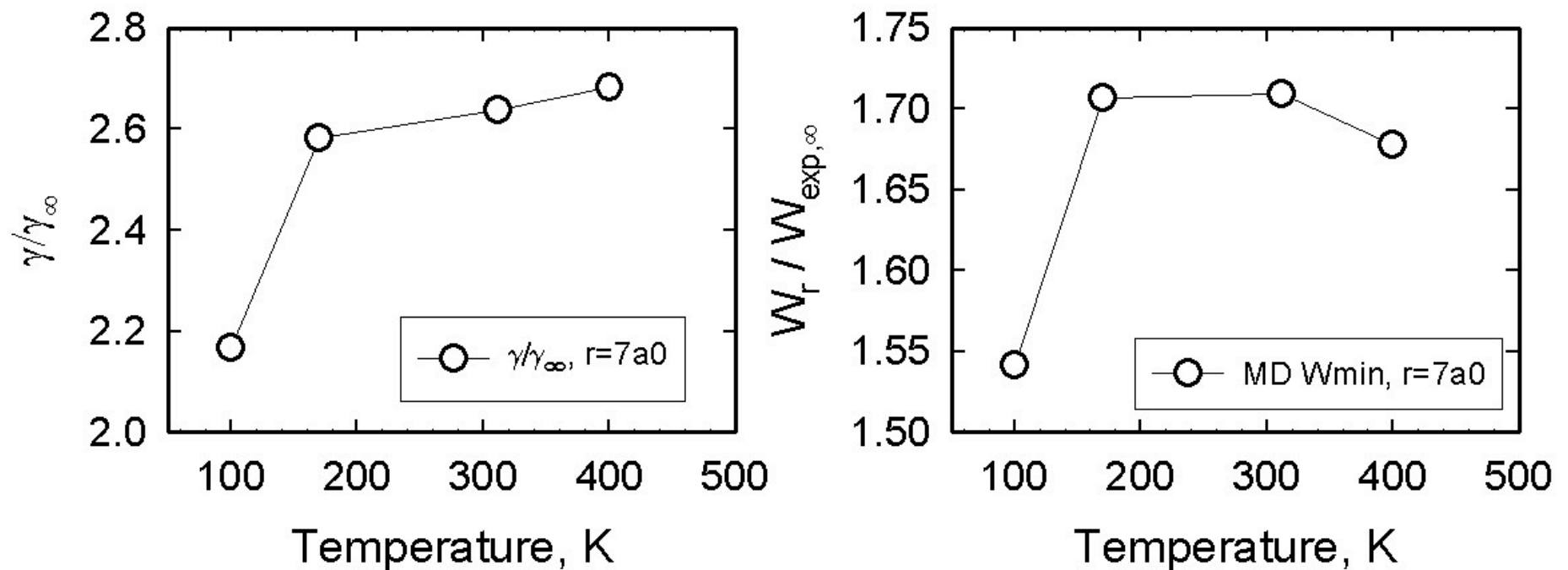
Comparison with the Tolman's formula



Fitting MD data to the Gibbs's formula



# Temperature dependence of $g$ , $W_{\min}$



$$W_{\exp} = \frac{4\pi}{3} R_s^3 \cdot \gamma_\infty$$

# Summary

- The new H-Li potential was tested by calculating the diffusion constants of H, D, and Tr in liquid Li and our results show that the tritium data agree well with experiment of Buxbaum & Johnson, 1985.
- Formation of small Hydrogen clusters was observed by this MD study for the first time. The relative amount of dimers is >10% at 480K.
- H, D, Tr/Li sputtering yield was calculated for the ions energy interval 10-300eV. At an energy below 50eV, the MD data are 3-10 times larger than the SRIM results.
- Cavity formation in liquid Li was studied by MD. Surface tension of small cavities was found to be larger than the experimental value for the plane surface by a factor of 2-3.
- Free energy of cavity formation in liquid lithium was calculated for a few cavity radii and they show that the cavity formation is allowed by thermodynamics.